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# EFFECT OF INTERPHASE TANGENTIAL FORCES ON FLOW DEVELOPMENT IN A WEAKLY CONDUCTIVE LIQUID

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Coulomb forces induce motion in a weakly conductive polarizable liquid by means of volume forces [1-5] and tangential surface stresses [2, 6]. While the former type of flow has a threshold character [1-5], the latter can develop in a vanishingly small electric field upon motion of surface charge over the free surface of the liquid [6]. The surface charge accumulation time on the free surface is of the order of magnitude of the free charge relaxation time $t_e = \varepsilon/\sigma$ [7]. If the problem characteristic time $t_0$ satisfies the inequality $t_0 \ll t_e$, then surface charge can be neglected and the major role will be played by polarization forces (for example, in problems involving stabilization of the free surface of a dielectric liquid by an electric field [2, 8, 9]). For $t_0 \geqslant t_e$ Coulomb surface forces cannot be ignored, and their consideration leads to the possibility of electroconvective flows.

In the present study the basic principles of thresholdless electroconvection will be considered, using the example of flow of a weakly conductive polarizable liquid under the action of surface forces produced by a special electrode geometry.

1. Formulation of the Problem. We will consider two incompressible viscous weakly conductive polarizable immiscible liquids, situated between two infinite horizontal electrodes and separated by a free surface S. We introduce a cartesian coordinate system as shown in Figs. 1, 2, and denote by $\Omega_i$ the region occupied by liquids, with $S_1 = (-\infty < x < \infty,\ z = h_1 + a\cos\omega x)$ being the upper curved electrode, and $S_2 = (-\infty < x < \infty,\ z = -h_2)$, the lower planar electrode. Here and below, the indices 1 and 2 refer to quantities defined in the regions $\Omega_1$, $\Omega_2$.

Liquid motion will be described by the electrohydrodynamics equations

$$\begin{gathered}\rho_i(\partial\mathbf{v}_i/\partial t + (\mathbf{v}_i\nabla)\mathbf{v}_i) = -\nabla p_i + \eta_i\Delta\mathbf{v}_i + q_i\mathbf{E}_i - \rho_i g\mathbf{e}_z,\\ \operatorname{div}\mathbf{v}_i = 0,\ \operatorname{div}\varepsilon_i\mathbf{E}_i = 4\pi q_i,\ \mathbf{E}_i = -\nabla\varphi_i,\\ \partial q_i/\partial t + \operatorname{div}\mathbf{j}_i = 0 \text{ on } \Omega_i,\end{gathered} \tag{1.1}$$

where $\rho_i$ is the density; $p_i$ is total pressure [8]: $\mathbf{j}_i = \sigma_i\mathbf{E}_i + q_i\mathbf{v}_i$ is current density; $\eta_i$, $\sigma_i$ are constant dynamic viscosity and conductivity coefficients; $q_i$ is volume charge density; $\varphi_i$ is electric field potential; $\varepsilon_i$ is dielectric permittivity; g is acceleration of gravity (i = 1, 2).

The boundary conditions for Eq. (1.1) follow from the conditions of adhesion, specification of potential on the electrodes, and kinematic, dynamic, and electrodynamic conditions on the free surface. They have the form [6, 8]

$$\begin{gathered}S_1 : \mathbf{v}_1 = 0,\ \varphi_1 = U = \text{const};\ S_2 : \mathbf{v}_2 = 0,\ \varphi_2 = 0;\\ S : \langle\mathbf{v}\rangle = 0,\ \partial f/\partial t = \mathbf{v}_1\cdot\mathbf{n}|\nabla f|^{1/2},\ \varphi_1 = \varphi_2,\\ \langle\varepsilon\mathbf{E}\cdot\mathbf{n}\rangle = 4\pi q_S,\ \partial q_S/\partial t + \operatorname{div}_S\mathbf{i} - Hq_S\mathbf{v}_1\cdot\mathbf{n} + \langle\mathbf{jn}\rangle = 0,\end{gathered}$$

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$$-\langle p\rangle + \langle \tau_{ik}\rangle n_i n_k + \langle T_{ik}\rangle n_i n_k = -2\alpha H,$$
$$\langle \tau_{ik}\rangle n_i \tau_k + q_S E_\tau = 0, \ \mathbf{i} = \sigma_S \mathbf{E}_\tau + q_S \mathbf{v}_\tau, \tag{1.2}$$

where $\mathrm{div}_s$ is the surface divergence; $\mathbf{i}$ is the surface current; $\sigma_s$ is the surface conductivity coefficient; $\mathbf{E}_\tau$, $\mathbf{v}_\tau$ are tangential components of the vectors $\mathbf{E}$, $\mathbf{v}$ on the surface S; $\mathbf{n} = (n_i)$, $\tau = (\tau_i)$ are normal and tangential unit vectors on S; H is the mean curvature of S; $\alpha$ is a constant surface tension coefficient; $q_S$ is the surface charge density; $T_{ik} = \varepsilon E_i E_k/4\pi - \varepsilon E^2\delta_{ik}/8\pi$ is the Maxwell stress tensor without striction components [8]; $\tau_{ik} = \eta(\partial v_i/\partial x_k + \partial v_k/\partial x_i)$ is the viscous stress tensor; $\langle F\rangle = F_1 - F_2$ is the change in a certain quantity F upon transition through the surface S; $z = f(t, x, y)$ is the equation of the surface S.

2. Solution of the Stationary Problem. In order to find the most significant features of electroconvection, we will assume that the curvature of the upper electrode is low, i.e., assume that the characteristic radius of curvature is significantly greater than the thicknesses $h_1$, $h_2$ and the inflection amplitude $a$:

$$\mu \sim a^2\omega^2 \sim ah_1\omega^2 \sim ah_2\omega^2 \ll 1.$$

In this case, the problem of Eqs. (1.1), (1.2) can be solved by perturbation theory methods, using the representation

$$\mathbf{v}_i = \mathbf{v}_{i1} + \mathbf{v}_{i2} + \ldots, \qquad p_i = p_{i0} + p_{i1} + p_{i2} + \ldots,$$
$$\varphi_i = \varphi_{i0} + \varphi_{i1} + \varphi_{i2} + \ldots, \quad f = f_1 + f_2 + \ldots,$$

where the terms $\mathbf{v}_{ik}, p_{ik}, \varphi_{ik}, f_k$ $(i = 1, 2)$ are of k-th order smallness in $\mu$, and $\varphi_i$, $p_{i0}$ is the solution describing the field potential and pressure distributions with a planar upper electrode.

We find

$$\varphi_{i0} = a_{i1}z + a_{i2}, \ p_{i0} = c_i - \rho_i g z, \ a_{i2}, \ c_i = \text{const},$$
$$a_{11} = \sigma_2 U/\gamma, \ a_{21} = \sigma_1 U/\gamma, \ \gamma \equiv \sigma_1 h_2 + \sigma_2 h_1 \ (i = 1, 2).$$

For the first approximation we obtain the problem

$$\begin{gathered}
-\nabla p_{i1} + \eta_i \Delta \mathbf{v}_{i1} = 0, \ \mathrm{div}\, \mathbf{v}_{i1} = 0, \ \Delta\varphi_{i1} = 0 \text{ on } D_i, \\
\mathbf{v}_{i1} = (v_{xi1}, v_{zi1}, 0) \quad (i = 1, 2), \\
D_1 = (-\infty < x < \infty, 0 \leqslant z \leqslant h_1), \ D_2 = (-\infty < x < \infty, \\
-h_2 \leqslant z \leqslant 0); \\
\text{for} \quad z = h_1 \ \mathbf{v}_{11} = 0, \ \varphi_{11} = -aa_{11}\cos\omega x, \\
\text{for} \quad z = -h_2 \ \mathbf{v}_{21} = 0, \ \varphi_{21} = 0, \\
\text{for} \ z = 0 \ v_{z11} = v_{z21} = 0, \ v_{x11} = v_{x21}, \ \varphi_{11} - \varphi_{21} = (a_{21} - a_{11})f_1, \\
\sigma_1 \frac{\partial\varphi_{11}}{\partial z} - \sigma_2 \frac{\partial\varphi_{21}}{\partial z} + \sigma_S \frac{\partial^2}{\partial x^2}(\varphi_{11} + a_{11}f_1) - q_{S0}\frac{\partial v_{x11}}{\partial x} = 0, \\
\alpha f_1'' + g\langle\rho\rangle - (p_{11} - p_{21}) + 2\frac{\partial}{\partial z}(\eta_1 v_{z11} - \eta_2 v_{z21}) + \frac{1}{4\pi}\frac{\partial}{\partial z}(\varepsilon_1 a_{11}\varphi_{11} - \\
-\varepsilon_2 a_{21}\varphi_{21}) = 0, \\
\frac{\partial}{\partial z}(\eta_1 v_{x11} - \eta_2 v_{x21}) - q_{S0}\frac{\partial}{\partial x}(\varphi_{21} + a_{21}f_1) = 0.
\end{gathered} \tag{2.1}$$

Here $q_{S0} = -(\varepsilon_1\sigma_2 - \varepsilon_2\sigma_1)U/4\pi\gamma$, and the prime denotes differentiation with respect to x.

Transforming to the flow function

$$v_{xi1} = -\partial\psi_i/\partial z, \ v_{zi1} = \partial\psi_i/\partial x \ (i = 1, 2)$$

and eliminating the pressure, from Eq. (2.1) we obtain

$$\Delta^2\psi_i = 0, \ \Delta\varphi_{i1} = 0 \text{ on } D_i \ (i = 1, 2); \tag{2.2}$$

$$\text{for} \quad z = h_1 \ \psi_1 = A_1, \ \partial\psi_1/\partial z = 0, \ \varphi_{11} = -aa_{11}\cos\omega x; \tag{2.3}$$

$$\text{for} \quad z = -h_2 \ \psi_2 = A_1, \ \partial\psi_2/\partial z = 0, \ \varphi_{21} = 0; \tag{2.4}$$

$$\text{for } z=0 \ \psi_1=\psi_2=A_1,\ \partial\psi_1/\partial z=\partial\psi_2/\partial z,\ \varphi_{11}-\varphi_{21}= = (a_{21}-a_{11})f_1; \tag{2.5}$$

$$\sigma_1\frac{\partial\varphi_{11}}{\partial z}-\sigma_2\frac{\partial\varphi_{21}}{\partial z}+\sigma_S\frac{\partial^2}{\partial x^2}(\varphi_{11}+a_{11}f_1)+q_{S0}\frac{\partial^2}{\partial x\partial z}\psi_2=0; \tag{2.6}$$

$$\left(\alpha f_1''+\langle\rho\rangle g f_1\right)''+\left(\Delta\frac{\partial^2}{\partial x\partial z}+2\frac{\partial^4}{\partial z\partial x^3}\right)(\eta_1\psi_1-\eta_2\psi_2)+ + \frac{1}{4\pi}\frac{\partial^3}{\partial x^2\partial z}(\varepsilon_1 a_{11}\varphi_{11}-\varepsilon_2 a_{21}\varphi_{21})=0; \tag{2.7}$$

$$\frac{\partial^2}{\partial z^2}(\eta_1\psi_1-\eta_2\psi_2)+q_{S0}\frac{\partial}{\partial x}(\varphi_{21}+a_{21}f_1)=0, \tag{2.8}$$

where $A_1$ = const.

Expanding the solution in the form

$$\varphi_{11}=\frac{1}{\operatorname{sh}\omega h_1}(-aa_{11}\operatorname{sh}\omega z+A_3G_1(z))\cos\omega x,$$
$$\varphi_{21}=A_4\frac{G_2(z)}{G_{20}}\cos\omega x,\quad f_1=A_2\cos\omega x,$$
$$\psi_1=A_5\left(\frac{zG_1(z)}{\omega h_1}-(h_1-z)\frac{\operatorname{sh}\omega z}{G_{10}}\right)\sin\omega x+A_1,$$
$$\psi_2=A_6\left(\frac{zG_2(z)}{\omega h_2}-(h_2+z)\frac{\operatorname{sh}\omega z}{G_{20}}\right)\sin\omega x+A_1,$$
$$G_1(z)=\operatorname{sh}\omega(h_1-z),\ G_2(z)=\operatorname{sh}\omega(h_2+z),$$
$$G_{10}=G_1(0),\ G_{20}=G_2(0),$$

we find that Eq. (2.2), conditions (2.3), (2.4), and the first condition of Eq. (2.5) are satisfied identically, while the remaining condition at z = 0 gives the following linear inhomogeneous system of fifth order equations for definition of the coefficients $A_i$ (i = 2, 3, ..., 6):

$$\begin{gathered}
A_5\left(\frac{G_{10}}{\omega h_1}-\frac{\omega h_1}{G_{10}}\right)-A_6\left(\frac{G_{20}}{\omega h_2}-\frac{\omega h_2}{G_{20}}\right)=0,\\
A_3-A_4-(a_{21}-a_{11})A_2=0,\\
(\sigma_S\omega+\sigma_1\operatorname{cth}\omega h_1)A_3+\sigma_2\operatorname{cth}\omega h_2\cdot A_4+\sigma_S a_{11}\omega A_2-\\
-q_{S0}\left(\frac{G_{20}}{\omega h_2}-\frac{\omega h_2}{G_{20}}\right)A_6=-\frac{aa_{11}\sigma_1}{G_{10}},\\
(\alpha\omega^2-\langle\rho\rangle g)A_2+2\omega\left(\eta_1\frac{\omega h_1}{G_{10}}A_5-\eta_2\frac{\omega h_2}{G_{20}}A_6\right)+\\
+\frac{\omega}{4\pi}(\varepsilon_1 a_{11}\operatorname{cth}\omega h_1 A_3+\varepsilon_2 a_{21}\operatorname{cth}\omega h_2\cdot A_4)=-\frac{aa_{11}^2\varepsilon_1\omega}{4\pi G_{10}},\\
2\eta_1\left(\frac{\operatorname{ch}\omega h_1}{\omega h_1}-\frac{1}{G_{10}}\right)A_5+2\eta_2\left(\frac{\operatorname{ch}\omega h_2}{\omega h_2}-\frac{1}{G_{20}}\right)A_6+q_{S0}(A_4+a_{21}A_2)=0.
\end{gathered} \tag{2.9}$$

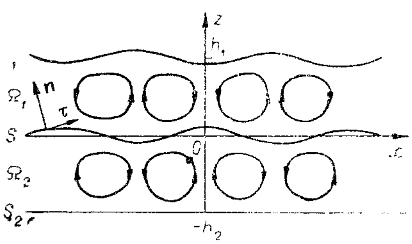


Fig. 1

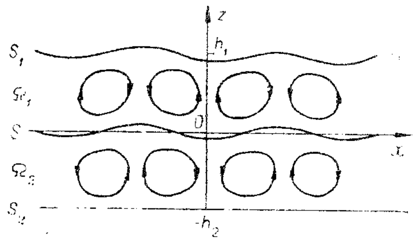


Fig. 2

This system of equations has a solution when and only when its determinant is nonzero. In the future we will consider the physical meaning of a zero determinant in greater detail, but meanwhile we will assume that it is nonzero. In this case the solution of system (2.9) has the form

$$\begin{gathered}
A_2=\frac{a\omega\sigma_2U^2}{4\pi\gamma^2G_{10}}\,\frac{\sigma_1a_1-\varepsilon_1\sigma_2a_2}{D}\quad(\gamma\equiv\sigma_1h_2+\sigma_2h_1),\\
A_6=q_{S0}\frac{a\omega h_2\sigma_1\sigma_2G_{20}U}{\gamma G_{10}D}\left[\alpha\omega^2-\langle\rho\rangle g+\frac{\omega\left(\varepsilon_1\sigma_2^2-\varepsilon_2\sigma_1^2\right)\operatorname{cth}\omega h_2U^2}{4\pi\gamma^2}\right],\\
A_4=-a_{21}A_2\frac{b}{\omega h_2G_{20}}A_6,\quad A_3=A_4+(a_{21}-a_{11})A_2,\\
b\equiv\eta_1(\operatorname{sh}2\omega h_1-2\omega h_1)\frac{G_{20}^2-\omega^2h_2^2}{G_{10}^2-\omega^2h_1^2}+\eta_2(\operatorname{sh}2\omega h_2-2\omega h_2),\\
A_5=\frac{h_1G_{10}}{h_2G_{20}}\,\frac{G_{20}^2-\omega^2h_2^2}{G_{10}^2-\omega^2h_1^2}A_6,\\
D=(\alpha\omega^2-\langle\rho\rangle g)\,a_2-\left[\left(\varepsilon_1\sigma_2^2\operatorname{cth}\omega h_1+\varepsilon_2\sigma_1^2\operatorname{cth}\omega h_2\right)a_2-\right.\\
\left.-\sigma_1\sigma_2(\operatorname{cth}\omega h_1+\operatorname{cth}\omega h_2)\,a_1\right]\omega U^2/4\pi\gamma^2,\\
a_1=(\varepsilon_1\sigma_2\operatorname{cth}\omega h_1+\varepsilon_2\sigma_1\operatorname{cth}\omega h_2)\,b+2\omega^2(\varepsilon_1\sigma_2-\varepsilon_2\sigma_1)\times\\
\times\left[\frac{\eta_1h_1^2\left(G_{20}^2-\omega^2h_2^2\right)}{G_{10}^2-\omega^2h_1^2}-\eta_2h_2^2\right],\\
a_2=(\sigma_S\omega+\sigma_1\operatorname{cth}\omega h_1+\sigma_2\operatorname{cth}\omega h_2)\,b+q_{S0}^2\left(\operatorname{sh}^2\omega h_2-\omega^2h_2^2\right).
\end{gathered}\tag{2.10}$$

3. Analysis of the Solution. From the solution obtained it is evident that electroconvective motion is absent ($A_5 = A_6 = 0$) in the following cases: 1) if the conductivity of even one liquid is equal to zero ($\sigma_1 = 0$ or $\sigma_2 = 0$); 2) if surface charge is absent from the free surface: $\varepsilon_1\sigma_2 - \varepsilon_2\sigma_1 = 0$; 3) if the surface conductivity $\sigma_S$ is sufficiently high: $\sigma_S \to \infty$; 4) if either one of the liquids is an ideal conductor ($\sigma_1 = \infty$, $\sigma_2 =$ const, or $\sigma_2 = \infty$, $\sigma_1 =$ const).

Development of electroconvective cells is explained by the fact that in motion of surface charge along the tangential field component on the free surface there arise tangential stresses

$$\tau_{n\tau}=\left\langle\eta\left(\frac{\partial v_i}{\partial x_k}+\frac{\partial v_k}{\partial x_i}\right)n_i\tau_k\right\rangle=-q_SE_\tau,\tag{3.1}$$

which lead to flow development. Such a type of motion has been observed in experiments with plane electrodes, one of which was constructed in a manner such that the potential varied periodically along the x axis [6].

Thus, motion of surface charge along the free surface can be one cause of thresholdfree electroconvection. From this it is clear that the phenomenon must be considered in analyzing electrohydrodynamic motions in weakly conductive liquids with a free surface located in inhomogeneous electric fields, for example, in a system of coaxial cylindrical electrodes [10] or a system with edge-plane type electrodes [4].

It is obvious that with sufficiently high voltage on the electrodes the expression for D, which coincides to the accuracy of a nonzero cofactor with the determinant of system (2.9), tends to zero. The solution obtained then increases without limit, which implies instability of the surface S. In fact, in view of the small curvature of the upper electrode it may be considered planar in a first approximation. If we consider the stability of the surface S with a planar upper electrode with respect to normal perturbations (x-dependence proportional to exp(ikx), where k is a wave number related to the length of the perturbation by the expression $\lambda = 2\pi/k$), then the critical voltage at which the surface S becomes unstable is determined by the condition of equality to zero of the expression for $D = D(\omega, U, \sigma_1, \sigma_2, \ldots)$, with the quantity $\omega$ replaced by k. Thus as soon as the voltage U reaches the value U(k) defined from the equation

$$D(\omega,\ U,\ \sigma_1,\ \sigma_2,\ \ldots)|_{\omega=k}=0,$$

and the perturbation wavelength coincides with the inflection period of the upper electrode $\omega = k$, a unique "resonance" develops and the surface S loses stability. We note that the surface can also lose stability earlier, since the critical voltage $U_*$, defined as

$$U_* = \min_{k>0} U(k),$$

may be less than $U(k)|_{k=\omega}$.

In the voltage range $U < U_*$, where the approximation considered here is valid, the sign of D is positive ($D > 0$). The form of the free surface will then be determined by the sign of the expression $B = \sigma_1 a_1 - \varepsilon_1 \sigma_2 a_2$. For example if $B > 0$ ($B < 0$), the depressions (projections) of the surface will be located opposite the projections of the upper electrode. The direction of rotation in the electroconvective cells will then be determined only by the signs of the expressions $\varepsilon_1\sigma_2 - \varepsilon_2\sigma_1$, $\varepsilon_1\sigma_2^2 - \varepsilon_2\sigma_1^2$.

We will consider some limiting cases, assuming for simplicity that the layer thicknesses are identical: $h_1 = h_2 = h$.

Let the conductivity of the lower layer of liquid be significantly greater than that of the upper layer: $\sigma_2 \gg \sigma_1$. Such is the case, for example, with benzene and water. Maintaining terms up to first order smallness in the small parameter $\sigma_1/\sigma_2$ in the expressions for the coefficients, we will have

$$A_2 = -\frac{\alpha\omega\varepsilon_1 E^2}{4\pi \operatorname{sh}\omega h D_-}, \quad A_6 = -\frac{\alpha\omega h\varepsilon_1\sigma_1 E^2 D_+}{a_2 D_-},$$
$$D_\pm = \alpha\omega^2 - \langle\rho\rangle g \pm \varepsilon_1\omega \operatorname{cth} \omega h E^2/4\pi,$$
$$A_5 = A_6,\ A_3 = -EA_2,\ A_4 = O(\sigma_1/\sigma_2),$$

where

$$a_2 = (\sigma_S\omega + \sigma_2 \operatorname{cth}\omega h)(\operatorname{sh} 2\omega h - 2\omega h)(\eta_1 + \eta_2) + \varepsilon_1^2 E^2(\operatorname{sh}^2\omega h - \omega^2 h^2)/16\pi^2;$$

$E \equiv U/h$; $O(\sigma_1/\sigma_2)$ is the order symbol.

The critical voltage for the present case is given by

$$E_* = U_*/h = \min_{k>0}\left[\frac{4\pi(\alpha k^2 - \langle\rho\rangle g)}{\varepsilon_1 k \operatorname{cth} kh}\right]^{1/2}.$$

For $E < E_*$ the coefficient $A_2$ is negative, whence it follows that the form of the free surface is "out of phase" with the form of the upper electrode; i.e., the projections of the free surface are located opposite the projections of the upper electrode (see Fig. 1). Such surface form can be explained by the fact that beneath the projections of the curved electrode on the free surface there are maxima in surface charge concentration, so that these segments are attracted more strongly than the segments beneath the depressions of the curved electrode. The character of the flow is then such that at the points where the free surface has projections ($\cos\omega x < 0$) the lower liquid flows downward and the upper liquid upward, while at the surface depressions ($\cos\omega x > 0$) the opposite is true, the lower liquid flowing upward, and the upper downward (see Fig. 1).

In the other limiting case $\sigma_1 \gg \sigma_2$ we have

$$A_2 = \frac{a\omega\varepsilon_2\sigma_2 E^2}{4\pi\operatorname{sh}\omega h}\,\frac{b\operatorname{cth}\omega h - 2\omega^2 h^2(\eta_1 - \eta_2)}{(\alpha\omega^2 - \langle\rho\rangle g - \varepsilon_2\omega\operatorname{cth}\omega h E^2/4\pi)\,a_2},$$

$$A_6 = \frac{a\omega h\varepsilon_2\sigma_2 E^2}{4\pi a_2},\quad A_5 = A_6,\quad A_3 = A_4 + EA_2,\quad A_4 = -EA_2 - \frac{bA_6}{\omega h\operatorname{sh}\omega h},$$

where

$$b = (\eta_1 + \eta_2)(\operatorname{sh} 2\omega h - 2\omega h);$$

$$a_2 = (\sigma_S\omega + \sigma_1\operatorname{cth}\omega h)\,b + \varepsilon_2^2 E^2(\operatorname{sh}^2\omega h - \omega^2 h^2)/16\pi^2,$$

and in this case at $b \coth \omega h - 2\omega^2 h^2(\eta_1 - \eta_2) > 0$ the coefficient $A_2$ is positive, so that we see a pattern opposite to that discussed above: the free surface is "in phase" with the curved electrode, and rotation in the electroconvective cells occurs in the opposite direction (Fig. 2).

We will now consider peculiarities of electroconvection at various limiting values of the liquid viscosities.

For vanishingly small $\eta_1$, $\eta_2$ the balance of forces, Eq. (3.1), would appear to lead to an abrupt increase in flow velocity. However at $\eta_1, \eta_2 \to 0$ the coefficients of the solutions are finite and have the form of Eq. (2.10), where we must take

$$a_1 = 0, \quad a_2 = q_{S0}^2 \left(\mathrm{sh}^2\, \omega h_2 - \omega^2 h_2^2\right), \quad b = 0.$$

Thus, for small viscosity coefficients $\eta_1$, $\eta_2$ the stationary velocity field is independent of liquid viscosity. This is explained by the fact that as $\eta_1, \eta_2 \to 0$ the tangential field component $E_\tau$ in Eq. (3.1) also tends to zero, maintaining the same order of smallness as the right side of Eq. (3.1).

In the case where either of the liquids has a sufficiently high viscosity, the flow velocity will decrease in inverse proportion to the dynamic viscosity coefficient, since as $\eta_1 \to \infty$ ($\eta_2 \to \infty$) the coefficients $A_5$, $A_6$ decrease as $\eta_1^{-1}$ ($\eta_2^{-1}$).

The solution takes on its simplest form when the surface tension or force of gravity is so high that the surface is undeformed, i.e., in the expressions for D in Eq. (2.10) the terms containing the parameters $\alpha$, g, are the major ones. In this case we have

$$A_2 = 0, \quad A_3 = A_4 = -\frac{bA_6}{\omega h_2 G_{20}}, \quad A_6 = \frac{a(\varepsilon_2\sigma_1 - \varepsilon_1\sigma_2)\sigma_1\sigma_2\omega h_2 G_{10} U^2}{a_2(\sigma_1 h_2 + \sigma_2 h_1)^2 G_{20}},$$

where b, $a_2$ are defined by Eq. (2.10) and $A_5$ is expressed in terms of $A_6$ by Eq. (2.9). Hence it is evident that the direction of rotation in the electroconvective cells is determined solely by the sign of the expression $\varepsilon_2\sigma_1 - \varepsilon_1\sigma_2$, i.e., by the sign of the surface charge. We will express $A_6$ as a function of the conductivities $\sigma_1$, $\sigma_2$ in explicit form, assuming for simplicity that $\sigma_S = 0$, $h_1 = h_2 = h$;

$$A_6 = A_6(\sigma_1, \sigma_2) = \frac{c_1\sigma_1\sigma_2(\varepsilon_2\sigma_1 - \varepsilon_1\sigma_2)}{(\sigma_1 + \sigma_2)^2[(\sigma_1 + \sigma_2)c_2 + c_3]},$$

$$c_1 = a\omega U^2/h, \quad c_2 = (\eta_1 + \eta_2)(\mathrm{sh}\, 2\omega h - 2\omega h)\, \mathrm{cth}\, \omega h, \quad c_3 = q_{S0}^2(\mathrm{sh}^2\omega h - \omega^2 h^2).$$

If we study this function at its extremum, we find that the flow velocity will be maximal for the following relationship between the conductivities:

$$\sigma_1 = \left[(\varepsilon_1 + \varepsilon_2) \pm \sqrt{(\varepsilon_1 + \varepsilon_2)^2 - \varepsilon_1\varepsilon_2}\right]\sigma_2/\varepsilon_2 \equiv \sigma_\pm. \tag{3.2}$$

The double sign on the right side of this equation corresponds to the different rotation directions in the electromagnetic cells. We will consider $\sigma_2$ fixed and study the dependence of $A_6$ on $\sigma_1$. As is evident from Fig. 3, at small $\sigma_1$ the flow velocity is also small. With increase in $\sigma_1$ $A_6 < 0$, and the velocity increases and reaches a maximum value at $\sigma_1 = \sigma_-$, which is defined by Eq. (3.2) with a minus sign. With further increase in $\sigma_1$ the flow velocity falls and goes to zero at $\sigma_1 = \sigma_0 \equiv \varepsilon_1\sigma_2/\varepsilon_2$, i.e., at zero surface charge. Further growth in $\sigma_1$ leads to the liquid again going into motion, but now with rotation in the cells

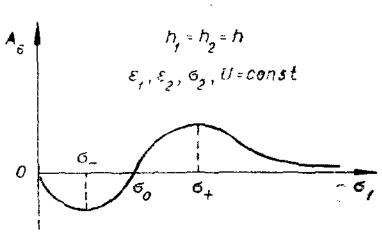


Fig. 3

in the opposite direction ($A_6 > 0$), and the rotation velocity reaches a maximum at $\sigma_1 = \sigma_+$, defined by Eq. (3.2) with the plus sign. It is obvious that similar principles will hold with variation of $\sigma_2$ and fixed $\sigma_1$.

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# PROPAGATION OF A CLEARING WAVE IN AN INHOMOGENEOUS COMBUSTIBLE AEROSOL


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The propagation of intense optical radiation in an aerosol is accompanied by clearing which develops due to the vaporization or (and) combustion of the aerosol particles. Induced clearing in fogs and clouds developing due to the vaporization of water drops in a powerful optical field has been the most fully studied up to now [1-4]. A decrease in the size of water particles leads to weakening of absorption, as a result of which clearing of the aerosol occurs. Peculiarities of the dynamics of clearing are due to the fact that the rate of particle combustion is not a unique function of the radiation intensity. The rate of particle combustion at a given time depends on the radiation intensity at previous times and, of course, depends on the character and type of chemical reactions taking place in the process of combustion. The dynamics of clearing in an inhomogeneous, monodisperse, combustible aerosol is analyzed in the present report.

1. It is known that the rate of heterogeneous combustion $K_S$ of a solid particle at a temperature $T$ below the ignition temperature $T_0$ can be taken as equal to zero ($T_0 \approx 1500$°K for carbon particles with a size of 1-10 μm). At $T > T_0$ the quantity $K_S$ is different from zero and, generally speaking, depends on $T$. If the radiation intensity is relatively low, then after ignition of a particle the heat released as a result of the chemical reaction of combustion will make the main positive contribution to its heat balance. Therefore, after the ignition of a particle the combustion rate can be considered as practically independent of the radiation intensity. In this case radiation plays the role of the initiator of combustion.

An elementary estimate of the time of heating a carbon particle with a characteristic size of ~1 μm to the ignition temperature determines a value of $\sim 10^{-5}$ sec. This time is much less than the other characteristic times of the given problem (for example, the characteristic time of burnup of a particle of the same size is $\sim 10^{-3}$ sec). Therefore, one can assume that a particle ignites practically instantly when a certain radiation intensity $I_0$ is reached at the given point. From the heat-balance equation we get the estimate


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